HEAT TRANSFER WITH FREE-CONVECTIVE TURBULENT FLOW ON A VERTICAL PERMEABLE SURFACE

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Heat transfer is calculated for uniform injection or suction in the case of natural convection on a vertical plate.

The work [1] theoretically and experimentally studied heat exchange in the case of turbulent natural convection with uniform injection or suction of a gas through a vertical permeable surface. The solution obtained was based on the use of an analogy between free-convective and forced motions. In the present work, we calculate heat transfer on the basis of a two-layer model of turbulent free-convective flow and obtain an approximate expression for the relative law of heat transfer on a permeable surface.

1. We will examine free-convective or mixed flow in a regime of thermogravitational turbulence generation, when turbulence is generated by buoyancy forces to a considerably greater extent than by a shift in mean velocity. For this regime, from the equality of the dissipative and thermogravitational terms in the turbulence energy balance equation [2-4]

$$\rho \frac{E^{3/2}}{l} = -C_1 \rho g \beta \overline{T' u_y'} = \frac{C_1 g \beta}{c_p} \lambda_t \frac{\partial T}{\partial y}$$

and with allowance for the relation of A. N. Kolmogorov for the turbulent thermal conductivity $\lambda_t = C_2 c_p \rho E^{1/2} l$, we obtain an expression for the turbulent heat flow

$$q_t = \lambda_t \frac{\partial T}{\partial y} = C (g\beta)^{1/2} c_p \rho l^2 \left(\frac{\partial T}{\partial y} \right)^{3/2}, \tag{1}$$

where $C = C_1^{1/2} C_2^{3/2}$.

With q_t = const and $l = y$, the well-known 1/3 law for the temperature distribution [3, 4] follows from Eq. (1).

In the presence of injection or suction, the heat flux distribution across the boundary layer will be assumed to have the form

$$q = q_w + c_p \rho V (T - T_w). \tag{2}$$

Equating (1) and (2) and integrating the resulting equation, we find the temperature distribution in the turbulent zone of the boundary layer.

$$\left[1 + \frac{c_p \rho V}{q_w} (T - T_w) \right]^{1/3} - \left[1 + \frac{c_p \rho V}{q_w} (T_1 - T_w) \right]^{1/3} = V \left(\frac{c_p \rho}{C^2 g \beta q_w} \right)^{1/3} \left(\frac{1}{y_1^{1/3}} - \frac{1}{y^{1/3}} \right). \tag{3}$$

The temperature distribution in the viscous sublayer has the form

$$T = T_w + \frac{q_w}{c_p \rho V} \left[\exp \left(\frac{c_p \rho V y}{\lambda} \right) - 1 \right]. \tag{4}$$

Allowing for (4) and assuming $y_1 \ll \delta$, Eq. (3) takes the following form with $y = \delta$:

$$\left[1 + \frac{c_p \rho V}{q_w} (T_e - T_w) \right]^{1/3} - \exp^{1/3} \left(\frac{c_p \rho V y_1}{\lambda} \right) = V \left(\frac{c_p \rho}{C^2 g \beta q_w y_1} \right)^{1/3} \tag{5}$$

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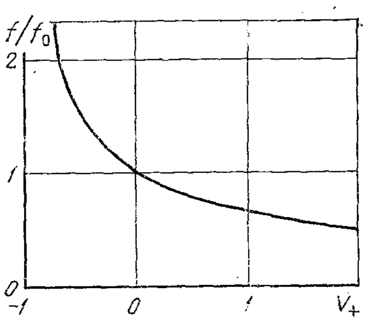

Fig. 1

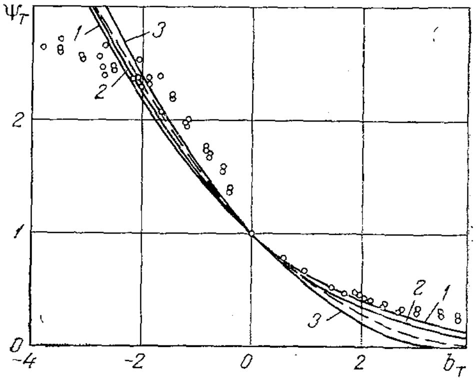

Fig. 2

Fig. 1. Change in the dimensionless thickness of the viscous sublayer with injection and suction.

Fig. 2. Dependence of the relative heat-transfer coefficient on the injection (suction) parameter: 1-3) Pr = 0.7; 1.0; 5.0.

On the basis of dimensional considerations, we assume the thickness of the viscous sublayer to be

$$y_1 = \frac{\nu}{u^*} f(V_+), \tag{6}$$

where $V_+ = V/u_*$ is a parameter representing the ratio of the convective force due to injection (suction) to the thermogravitational force; u_* is the analog of the dynamic velocity in the case of free convective flow.

We will introduce the dimensionless relative parameter

$$\Psi_T = \frac{q_w}{q_{w_0}} = \frac{\mathrm{Nu}}{\mathrm{Nu}_0}, \quad b_T = \frac{\mathrm{Pe}_V}{\mathrm{Nu}_0}, \tag{7}$$

where Nu_0 is the Nusselt number on an impermeable surface

$$\mathrm{Nu}_0 = \frac{\mathrm{Gr}_{q_0}^{1/4}}{f_0 + 3/C^{2/3}\mathrm{Pr}^{2/3} f_0^{1/3}}, \quad f_0 = f(V_+ = 0). \tag{8}$$

Comparison of the Nusselt numbers Nu and Nu_0 is done at the same value of the Grashof number $\mathrm{Gr}_{\Delta T}$, obtained from the temperature drop $\Delta T = T_e - T_w$; with allowance for (7), we write Eq. (5) in the form

$$\left(1 + \frac{b_T}{\Psi_T}\right)^{1/3} - \exp^{1/3}[\mathrm{Pr}\, V_+ f(V_+)] = V_+ \left(\frac{\mathrm{Pr}}{C^2 f(V_+)}\right)^{1/3}, \tag{9}$$

where $V_+ = b_T\,[(f_0 + 3/C^{2/3}\,\mathrm{Pr}^{2/3} f_0^{1/3})\,\mathrm{Pr}\,\Psi_T^{1/4}]^{-1}$.

The constants C and f_0 were chosen from comparison of the temperature distribution (3) and the formula for heat transfer (8) with the experimental data for a vertical nonpermeable surface. With selected values of the constants C = 0.975 and f_0 = 2.31, Eq. (8) agrees within the range $0.7 \leqslant \mathrm{Pr} \leqslant 5$ with the well-known formula $\mathrm{Nu}_0 = 0.13(\mathrm{Pr}\mathrm{Gr}_{\Delta T})^{1/3}$ to within 4%.

The effect of injection or suction rate on the thickness of the viscous sublayer was determined on the basis of assumption of the universality of the relation $\Delta_+/\Delta_{+0} = F(V_+\Delta_{+0})$ for forced and free-convective flows ($\Delta_+ = y_1 U_*/\nu$). In the case of forced convection, this relation was found by using the hypothesis of resistance of the viscous sublayer to random perturbations emanating from the turbulent core, i.e., by calculating the critical Reynolds number by an energy method [5]. The resulting function $f(V_+)$ in Eq. (6) is shown in Fig. 1.

Figure 2 shows the results of the use of Eq. (9) to calculate the dependence of the relative heat-transfer coefficient Ψ_T on the injection (suction) parameter b_T for different

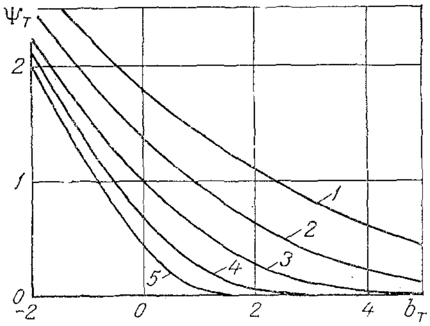

Fig. 3

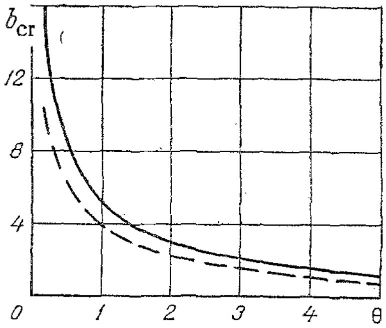

Fig. 4

Fig. 3. Effect of the temperature factor on the heat-transfer coefficient: 1-5) Θ = 0.25; 0.5; 1.0; 2.0; 4.0.

Fig. 4. Dependence of the critical injection parameter on the temperature factor.

Prandtl numbers (the solid lines). An increase in the rate of injection is accompanied by the asymptotic approach of the heat-transfer coefficient to zero. In the case of suction, the calculated values are valid up to the point of intersection with the straight line $\Psi_T = -b_T$, corresponding to the asymptotic solution of the energy equation with strong injection. The effect of the Prandtl number on the dependence of Ψ_T on b_T is qualitatively the same as in the case of forced convection [6]. It is clear from Fig. 2 that the calculated results agree satisfactorily with the experimental data for air [1] (points).

2. To calculate characteristics of the turbulent boundary layer with forced convection, wide use has been made of relative limiting laws of friction and heat transfer [7]. These limiting relations are based on the Prandtl (or Karman) law for turbulent shear stresses and heat flow in the case of approach of the Reynolds numbers to infinity, when the effect of the viscous sublayer can be ignored. Expressions with a similar significance can be obtained for a free-convective boundary layer. However, in contrast to forced flow, the relative laws for free convection may be only of an approximate nature, since the effect of the viscous sublayer on friction and heat exchange remains substantial at all Grashof numbers.

Integrating Eq. (1) across the boundary layer and assuming that the value of the integral is independent of factors complicating the flow, we obtain an approximate relative law of heat transfer for free-convective flow

$$\int_0^1 \left(\frac{\bar{c}_p \bar{\rho} \bar{q}_0}{\Psi_T \bar{q}}\right)^{2/3} d\bar{T} = 1. \tag{10}$$

As the simplest standard flow with which we can compare a more complex flow, we will examine a quasiisothermal flow with constant physical properties on a nonpermeable surface.

We will determine the effect of a change in density across the boundary layer on the heat transfer of a gas with a constant heat capacity, obeying the Clapeyron–Mendeleev equation of state $\rho/\rho_e = T_e/T$. Assuming

$$\bar{q} = \left(1 + \frac{b_T}{\Psi_T} \bar{T}\right) \bar{q}_0,$$

from (10) we obtain an expression for the law of heat transfer on a permeable surface with allowance for variable density

$$\Psi_T = \left\{ \int_0^1 \frac{d\bar{T}}{[(1 + B_T \bar{T})(\Theta + (1-\Theta)\bar{T})]^{2/3}} \right\}^{3/2}. \tag{11}$$

It follows from (11) for a nonpermeable plate ($B_T = 0$) that

$$\Psi_T = \left(\frac{3}{1 + \Theta^{1/3} + \Theta^{2/3}}\right)^{3/2}.$$

For a permeable plate, without allowance for variability of the physical properties ($\Theta = 1$), from (11) we obtain

$$\Psi_T = \left(\sqrt{1 - \frac{b_T}{108}} - \frac{b_T}{6} \right)^3. \tag{12}$$

It can be seen from Fig. 2 that Eq. (12) (the dashed line) satisfactorily agrees with the results of calculations with Eq. (9) when $Pr \approx 1$. This fact once more confirms the assertion made in [8] that, in determining relative integral parameters, a more thorough description of the process is possible than when determining absolute values of the same parameters. However, in contrast to the asymptotic character of the approach of the heat-transfer coefficient to zero which follows from (9), Eq. (12), as in the case of limiting laws for forced convection, leads to the appearance of a critical value of the injection parameter $b_{cr} = \sqrt{27}$, at which heat flow vanishes.

Figure 3 shows the effect of the temperature factor on the relation $\Psi_T(b_T)$. The value of b_{cr}, with allowance for the change in density across the boundary layer, is determined in accordance with (10) by the expression

$$b_{\rm cr} = \left\{ \int_0^1 \frac{d\overline{T}}{[\overline{T}(\Theta + (1-\Theta)\overline{T})]^{2/3}} \right\}^{3/2}.$$

The character of the dependence of b_{cr} on Θ (Fig. 4) turns out to be the same as in the case of forced convection [7] (dashed line), although the value of b_{cr} is higher in the case of free-convective flow (solid line).

NOTATION

u_x, u_y, components of velocity along and across the boundary layer; δ, thickness of the boundary layer; g, acceleration due to gravity; β, coefficient of cubical expansion; c_p, specific heat; $\Theta = T_w/T_e$, temperature factor; E, density of turbulence energy; l, scale of turbulence; V, rate of injection or suction; C_1, C_2, C, constants; $Gr_0 = g\beta q_w \delta^4/\lambda\nu^2$; $Gr_{\Delta T} = g\beta\Delta T\delta^3/\nu^2$, Grashof numbers; Nu, Nusselt number; Pr, Prandtl number; $\overline{c_p} = c_p/c_{p_e}$; $\overline{\rho} = \rho/\rho_e$; $\overline{q} = q/q_w$; $\overline{T} = (T - T_w)/\Delta T$; $B_T = b_T/\Psi_T$; $u_* = (g\beta q_w \nu^2/\lambda)^{1/4}$. Indices: w, wall; e, external flow; 0, impermeable surface; l, boundary of viscous sublayer; +, –, universal coordinates.

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